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D-branes in the NS5 Near-horizon pp-Wave Background

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Abstract

A class of D_p and Dp - $D(p+4)$ -brane solutions are constructed in the Penrose limit of the linear-dilaton geometry. The classical solutions are shown to break all space-time supersymmetries. In the worldsheet description, the branes preserve as many supersymmetries as D-branes in flat space. It is shown that these supersymmetries do not have zero modes on the worldsheet and hence do not admit local space-time realizations. This indicates that, unlike the perturbative spectrum, the D-brane spectrum of strings in the linear-dilaton pp-wave is not similar to the flat space case.

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1 Introduction

String theory in plane wave backgrounds that arise as Penrose limits of certain near-horizon geometries [1] is known to provide a holographic description of certain sectors of the dual field theory [2]. The Penrose limits of geometries of the form $AdS_p \times S^q$ lead to pp-wave/CFT correspondence and are of particular interest [2, 3, 4, 5]. The introduction of D-branes in the $AdS_p \times S^q$ and pp-wave space-times corresponds to considering defect conformal field theories on the dual side [6, 7, 8]. For D-brane classical solutions in such pp-wave backgrounds see also [9]-[17].

In this paper we are interested in a Penrose limit of the near horizon geometry of the NS5-brane (the linear-dilaton background) considered in [18, 19, 20]. String theory in this background is related, by a coordinate transformation, to the Nappi-Witten model [21] which is well studied (see, for example, [22, 23, 24]). In particular it is known that the worldsheet theory in the light-cone gauge can be mapped to a theory of strings in flat background by a simple field redefinition. However, as a result of the field redefinition, closed strings in the redefined theory have twisted boundary conditions that shift their oscillator frequencies.

In [20] it was argued that this pp-wave limit of the linear dilaton geometry contains the holographic description of a high-energy and large R-charge sector of the little string theory [25] that is dual to the linear dilaton space-time. The similarity of string theory in the linear-dilaton pp-wave to strings in flat background then suggested that the spectrum of little string theory in the high-energy and large R-charge regime is very similar to the spectrum of free strings in 10 dimensional flat space-time.

In this paper we consider D-branes embedded in the linear-dilaton pp-wave background. The purpose is to investigate to what extent the similarity with string theory in flat space-time persists. It is shown that significant differences arise in the way space-time supersymmetry is manifested. We consider a class of Dp -branes and $(Dp-Dp')$ -branes that contain the light-cone directions of the pp-wave within their worldvolumes but share the transverse directions of the pp-wave. It is shown that the classical solutions do not preserve any space-time supersymmetry. On the other hand, in the worldsheet description, these D-branes are shown to preserve the same amount of supersymmetry as the corresponding D-branes in flat space. However, the associated supercharges do not contain zero-mode pieces and therefore do not have local realizations in space-time. One expects this to have implications for the analogue of the defect CFT in the context of little string theories. For related work on open strings in NS-NS pp-wave background see [26, 27, 28].

The paper is organized as follows: In section 2, we present Dp and $(Dp-Dp')$ -brane solutions in the background obtained as a Penrose limit of the NS5-brane near-horizon geometry. In section 3, we show that these solutions break all space-time supersymmetries. In section 4, we describe the D1-brane in the light-cone worldsheet theory and show that while supersymmetry is preserved on the worldsheet it does not admit a local space-time realization. Section 5 contains our conclusions and discussions.

2 Classical solutions

In this section, we present classical solutions corresponding to Dp and Dp - Dp' branes in the plane wave background with constant NS-NS 3-form flux that arises as a Penrose limit of the near horizon geometry of the NS5-brane ([20]). The string frame metric, 3-form H and dilaton of the NS5-brane background are given by,

$$\begin{aligned} ds^2 &= -dt^2 + dy_5^2 + f(r)(dr^2 + r^2(d\theta^2 + \cos^2\theta d\psi^2 + \sin^2\theta d\phi^2)), \\ H &= N\epsilon_3, \quad e^{2\phi} = g_s^2 f(r), \quad f(r) = 1 + \frac{Nl_s^2}{r^2}, \end{aligned}$$

where ϵ_3 is the volume form on the transverse S_3 and N is the NS5-brane charge. The near-horizon geometry corresponds to the limit $r \rightarrow 0$ which removes the 1 in $f(r)$ and, on rescaling the time, leads to the linear dilaton background,

$$ds^2 = Nl_s^2(-d\tilde{t}^2 + \frac{dr^2}{r^2} + \cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\phi^2) + dy_5^2.$$

The Penrose limit is then taken with respect to the null geodesic along the equator ($\theta = 0$) of the transverse S^3 resulting in the following expressions for the metric and NS-NS 3-form [20],

$$\begin{aligned} ds^2 &= 2dx^+dx^- - \mu^2(z_1^2 + z_2^2)(dx^+)^2 + \sum_{a=1}^8 dz^a dz^a, \\ H_{+12} &= 2\mu. \end{aligned} \tag{2.1}$$

This is the background that we are interested in. It is related to the Nappi-Witten model [21] by a coordinate transformation and has been recently discussed in the context of pp-waves in [18, 19, 20]. In particular it was argued in [20] that string theory in this background provides a holographic description of a high-energy and high R-charge sector of little string theory [25]. The NS-NS 2-form corresponding to the above background will be taken as

$$B_{+1} = \mu z^2, \quad B_{+2} = -\mu z^1. \tag{2.2}$$

This is related to other choices of the B-field by 2-form gauge transformations which are symmetries of the closed string sector but modify worldvolume gauge fields in the case of open strings.

We start by writing down the D-string solution where the brane worldvolume is along the light-cone directions of the plane wave,

$$\begin{aligned} ds^2 &= f_1^{-\frac{1}{2}}[2dx^+dx^- - \mu^2 \sum_{i=1}^2 z_i^2(dx^+)^2] + f_1^{\frac{1}{2}} \sum_{a=1}^8 (dz^a)^2, \\ H_{+12} &= 2\mu, \quad F_{+-a} = \partial_a f_1^{-1}, \\ e^{2\phi} &= f_1, \quad f_1 = 1 + \frac{N_1 g_s l_s^6}{r^6}. \end{aligned} \tag{2.3}$$

For zero D-string charge this reduces to the pp-wave background (2.1) and for $\mu = 0$ it gives the D-string in flat space. We have checked explicitly that the above solution satisfies type-IIB field equations. Since the light-cone directions of the pp-wave are within the brane worldvolume, it is possible to construct this solution in the light-cone worldsheet theory in terms of open-string boundary conditions. We will see below that in spite of its simple form, this solution does not preserve any space-time supersymmetry. The worldsheet origin of the absence of supersymmetry will be discussed later.

Other Dp-branes for $2 \leq p \leq 7$ can be obtained by smearing the D-string along some of the transverse directions $z^3 \cdots z^8$ and applying T -dualities. Since the D-string solution (2.3) does not preserve supersymmetries, smearing the background is not *a priori* justifiable (since it is not obvious that stable periodic arrays could be built). However, one can check that the resulting configurations satisfy the IIA or IIB equations of motion, as the case may be. Then, for example, the $D3$ -brane solution in the NS5 pp-wave background is obtained by applying T -dualities along z^3 and z^4 -directions,

$$ds^2 = f_3^{-\frac{1}{2}} [2dx^+ dx^- - \mu^2 \sum_{i=1}^2 z_i^2 (dx^+)^2 + \sum_{\alpha=3}^4 (dz^\alpha)^2] + f_3^{\frac{1}{2}} \sum_{a=1,2,5}^8 (dz^a)^2, \\ H_{+12} = 2\mu, \quad F_{+-34a} = \partial_a f_3^{-1}, \quad e^{2\phi} = 1, \quad f_3 = 1 + \frac{N_3 g_s l_s^4}{r^4}, \quad (2.4)$$

with f_3 being the harmonic function in the transverse 6-space.

Now we present classical solutions for intersecting branes of type $(Dp - D(p+4))$ in the NS5 plane wave background. The classical solution for intersecting $(D1 - D5)$ -branes is given by,

$$ds^2 = (f_1 f_5)^{-\frac{1}{2}} [2dx^+ dx^- - \mu^2 \sum_{i=1}^2 z_i^2 (dx^+)^2] \\ + f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}} \sum_{\alpha=3}^6 (dz^\alpha)^2 + (f_1 f_5)^{\frac{1}{2}} \sum_{a=1,2,7,8} (dz^a)^2, \\ H_{+12} = 2\mu, \quad F_{+-a} = \partial_a f_1^{-1}, \quad F_{abc} = \epsilon_{abcd} \partial_d f_5, \\ e^{2\phi} = \frac{f_1}{f_5}, \quad f_{1,5} = 1 + \frac{N_{1,5} g_s l_s^2}{r^2}. \quad (2.5)$$

f_1, f_5 are harmonic functions in the common transverse directions (z^1, z^2, z^7, z^8) . We have checked that the above solution satisfies all type-IIB field equations.

Again, other solutions can be obtained from this by T -dualities, for example, the intersecting $(D3 - D7)$ -branes configuration,

$$ds^2 = (f_3 f_7)^{-\frac{1}{2}} [2dx^+ dx^- - \mu^2 \sum_{i=1}^2 z_i^2 (dx^+)^2 + \sum_{\alpha=3}^4 (dz^\alpha)^2] \\ + f_3^{\frac{1}{2}} f_7^{-\frac{1}{2}} \sum_{\beta=5}^8 (dz^\beta)^2 + (f_3 f_7)^{\frac{1}{2}} \sum_{i=1}^2 (dz^i)^2,$$

$$\begin{aligned}
H_{+12} &= 2\mu, & F_{+-34i} &= \partial_i f_3^{-1}, & \partial_i \chi &= \epsilon_{ij} \partial_j f_7 \\
e^{2\phi} &= f_7^{-2}, & f_{3,7} &= 1 + c \ln \frac{r}{l},
\end{aligned} \tag{2.6}$$

More intersecting brane solutions can be obtained by using T -dualities along z^3, \dots, z^8 directions in (2.5). D-branes intersecting at angle can also be obtained following [14, 29], however, we will skip these details.

3 Supersymmetry analysis of classical solutions

Let us now look at the supersymmetry of the D-brane solutions in the NS5 near-horizon plane wave by solving the type IIB Killing spinor equations. The supersymmetry variations of dilatino and gravitino fields of type IIB supergravity in the string frame are given by [30, 31],

$$\delta\lambda_{\pm} = \frac{1}{2}(\Gamma^{\mu}\partial_{\mu}\phi \mp \frac{1}{12}\Gamma^{\mu\nu\rho}H_{\mu\nu\rho})\epsilon_{\pm} + \frac{1}{2}e^{\phi}(\pm\Gamma^{\mu}F_{\mu}^{(1)} + \frac{1}{12}\Gamma^{\mu\nu\rho}F_{\mu\nu\rho}^{(3)})\epsilon_{\mp}, \tag{3.1}$$

$$\begin{aligned}
\delta\Psi_{\mu}^{\pm} &= \left[\partial_{\mu} + \frac{1}{4}(w_{\mu\hat{A}\hat{B}} \mp \frac{1}{2}H_{\mu\hat{A}\hat{B}})\Gamma^{\hat{A}\hat{B}}\right]\epsilon_{\pm} \\
&+ \frac{1}{8}e^{\phi}\left[\mp\Gamma^{\lambda}F_{\lambda}^{(1)} - \frac{1}{3!}\Gamma^{\lambda\nu\rho}F_{\lambda\nu\rho}^{(3)} \mp \frac{1}{2.5!}\Gamma^{\lambda\nu\rho\alpha\beta}F_{\lambda\nu\rho\alpha\beta}^{(5)}\right]\Gamma_{\mu}\epsilon_{\mp},
\end{aligned} \tag{3.2}$$

where μ, ν, ρ, λ are ten dimensional space-time indices, and hated indices refer to the Lorentz frame.

Note that for the background (2.1), the vanishing of the above supersymmetry variations leads to the Killing spinors,

$$\epsilon_{\pm}^{(bg)} = e^{\pm\frac{\mu}{2}x + \Gamma^{\hat{1}\hat{2}}}\epsilon_{\pm}^{(0)}, \quad \Gamma^{\hat{1}}\epsilon_{\pm}^{(0)} = 0, \tag{3.3}$$

where $\epsilon_{\pm}^{(0)}$ are constant spinors.

As for the D-brane solutions (2.3)-(2.6) in the background (2.1), it suffices to analyze the basic D-string solution (2.3). The vanishing of the dilatino variations give

$$\frac{f_{1,\hat{a}}}{f_1}\Gamma^{\hat{a}}(\epsilon_{\pm} - \Gamma^{\hat{1}\hat{2}}\epsilon_{\mp}) \mp 2\mu f_1^{-1/4}\Gamma^{\hat{1}\hat{2}}\epsilon_{\pm} = 0, \tag{3.4}$$

where hats denote Lorentz frame indices and $\hat{a} = 1 \dots 8$. On multiplying by $\Gamma^{\hat{1}}$ these reduce to $\Gamma^{\hat{1}}(\epsilon_{+} + \epsilon_{-}) = 0$. Also, adding the upper and lower sign equations in (3.4) leads to $\Gamma^{\hat{1}}(\epsilon_{+} - \epsilon_{-}) = 0$, while subtracting the two gives $\Gamma^{\hat{2}}(\epsilon_{+} - \epsilon_{-}) = 0$. The only non-trivial solution of these equations is

$$\epsilon_{+} = \epsilon_{-} = \epsilon, \quad \Gamma^{\hat{1}}\epsilon = 0. \tag{3.5}$$

The second condition is a common feature of pp-wave backgrounds, while the first is enforced by the presence of the D-string. The only component of the gravitini variations consistent with the above is ¹

$$\delta\psi_{-}^{\pm} \equiv \partial_{-}\epsilon_{\pm} + \frac{1}{8} \frac{f_{1,\hat{a}}}{f_1^{5/4}} \Gamma^{\hat{a}} \Gamma^{\hat{+}} \epsilon_{\mp} = 0, \quad (3.6)$$

giving $\partial_{-}\epsilon = 0$. Among the remaining gravitini variations let us first consider

$$\begin{aligned} \delta\psi_{+}^{\pm} &\equiv \partial_{+}\epsilon_{\pm} + \frac{1}{8} f_1^{-5/4} f_{1,\hat{a}} \Gamma^{\hat{a}} \left(\Gamma^{\hat{-}} + \frac{1}{2} \mu^2 z_i z_i \Gamma^{\hat{+}} \right) (\epsilon_{\pm} - \epsilon_{\mp}) \\ &\quad + \frac{1}{2} \mu^2 z_i f_1^{-1/4} \Gamma^{\hat{i}} \Gamma^{\hat{+}} \epsilon_{\pm} \mp \frac{\mu}{2} f_1^{-1/2} \Gamma^{\hat{i}\hat{2}} \epsilon_{\pm} = 0. \end{aligned} \quad (3.7)$$

On imposing (3.5), these reduce to the two equations $\partial_{+}\epsilon \mp \frac{\mu}{2} f_1^{-1/2} \Gamma^{\hat{i}\hat{2}} \epsilon = 0$ which are clearly inconsistent with each other. Even if one of the above two equations is relaxed, the solution $\epsilon(x^{+}, x^a)$ is not consistent with the vanishing of the remaining gravitini variations,

$$\delta\psi_c^{\pm} \equiv \partial_c \epsilon_{\pm} - \frac{1}{8} \delta_{cb} \frac{f_{1,\hat{a}}}{f_1^{3/4}} \left(\Gamma^{\hat{a}\hat{b}} \epsilon_{\pm} + \Gamma^{\hat{a}} \Gamma^{\hat{b}} \Gamma^{\hat{+}\hat{-}} \epsilon_{\mp} \right) \mp \frac{\mu}{2} \delta_{ci} \Gamma^{\hat{i}\hat{+}} \epsilon_{\pm} = 0, \quad (3.8)$$

due to the specific form of its x^{+} -dependence. This shows that the D-string solution in equation (2.3) does not preserve any of the space-time supersymmetries. Similarly one can show that, for the same reasons as above, the intersecting D-brane solutions in the NS5 pp-wave background presented in the previous section also do not preserve any space-time supersymmetry. This may sound a little puzzling since the worldsheet theory in the background (2.1) is closely related to the theory in flat space-time. Below we will see that the small difference between the two is enough to destroy all space-time supersymmetries in the D-brane sector.

4 Worldsheet analysis of D-brane supersymmetry

String propagation in the NS5 pp-wave background is related to the Nappi-Witten model [21] and has been extensively studied [19, 22, 23, 24, 26, 27, 28]. The worldsheet theory is greatly simplified in the light-cone Green-Schwarz description where by a field redefinition the action reduces to that of strings in flat space-time. The similarity of closed string spectrum in this pp-wave background to that in flat space-time has been emphasized in the context of the dual little string theory [20]. However, as we have seen above, at the level of classical solutions, the supersymmetry of D-brane excitations is very different from flat background. We now discuss this issue for the D-string solution from the worldsheet point of view. The other cases are similar.

¹In our conventions, $\Gamma^{\hat{+}\hat{-}} \Gamma^{\hat{+}} = -\Gamma^{\hat{+}} \Gamma^{\hat{+}\hat{-}} = \Gamma^{\hat{+}}$ and $\Gamma^{\hat{+}\hat{-}} \Gamma^{\hat{-}} = -\Gamma^{\hat{-}} \Gamma^{\hat{+}\hat{-}} = -\Gamma^{\hat{-}}$.

The light-cone gauge Green-Schwarz action for strings in the linear dilaton background (2.1) can be worked out using the IIB GS action in [32] or can be directly read off from [24], as²

$$\begin{aligned} \mathcal{S} = & \int d^2\sigma \left[\sum_{i=1}^2 (\partial_+ X^i \partial_- X^i - m^2 X^i X^i) - 2m (X^1 \partial_\sigma X^2 - X^2 \partial_\sigma X^1) \right. \\ & + \sum_{a=3}^8 \partial_+ X^a \partial_- X^a \\ & \left. + i (S_L^T \partial_- S_L + \frac{m}{2} S_L^T \gamma^{12} S_L) + i (S_R^T \partial_+ S_R - \frac{m}{2} S_R^T \gamma^{12} S_R) \right]. \quad (4.1) \end{aligned}$$

Here $S_{L,R}$ are $SO(8)$ spinors in, say, $\mathbf{8}_s$, $m = \mu p^+$ and $\partial_\pm = \partial_\tau \pm \partial_\sigma$. For closed strings the boundary conditions are periodicity in the X 's and $S_{L,R}$. For open strings, the boundary terms in the variation of the action can be set to zero by the D1-brane boundary conditions

$$\delta X^i|_{\sigma=0,\pi} = 0, \quad \delta X^a|_{\sigma=0,\pi} = 0, \quad S_L - \Omega S_R|_{\sigma=0,\pi}, \quad (\Omega^T \Omega = 1), \quad (4.2)$$

which are imposed for all τ . Ω is a constant matrix to be determined by the requirement that supersymmetry transformations keep the boundary conditions invariant.

The action (4.1) can be recast as a flat-space action by field redefinitions. In the bosonic sector the required transformation is a rotation by an angle $m\sigma$ [23],

$$Y^1 + iY^2 = e^{im\sigma} (X^1 + iX^2). \quad (4.3)$$

As for the fermions, there are two possible choices. For closed strings, where boundary conditions do not relate S_L and S_R , it is convenient to use

$$S_L = \exp(-\frac{m}{2}\tau\gamma^{12}) \hat{S}_L^{closed}, \quad S_R = \exp(\frac{m}{2}\tau\gamma^{12}) \hat{S}_R^{closed}. \quad (4.4)$$

Then, $\hat{S}_L^{closed}(\tau + \sigma)$ and $\hat{S}_R^{closed}(\tau - \sigma)$ satisfy the free field equations with periodic boundary conditions along σ . In the light-cone gauge, the zero mode part of these can be written entirely in terms of the space-time variable $X^+ = p^+ \tau$ which is consistent with the structure of Killing spinors of the NS5 pp-wave background (3.3).

However for open strings where (4.2) should hold for all τ , we find it convenient to define

$$\hat{S}_{L,R} = \exp(-\frac{m}{2}\sigma\gamma^{12}) S_{L,R}, \quad (4.5)$$

which is a spinor representation of (4.3). The action then takes the form

$$\mathcal{S} = \int d^2\sigma \left[\sum_{i=1}^2 \partial_+ Y^i \partial_- Y^i + \sum_{a=3}^8 \partial_+ X^a \partial_- X^a + i (\hat{S}_L^T \partial_- \hat{S}_L + \hat{S}_R^T \partial_+ \hat{S}_R) \right]. \quad (4.6)$$

²For the B -field we use the form (2.2) which, for closed strings, is equivalent to other choices related to it by 2-form gauge transformations, *e.g.*, $B_{12} = 2\mu x^+$. For open strings there is some ambiguity which we fix by using (2.2) and setting the worldvolume gauge fieldstrength to zero. This is consistent with our classical solution and allows fixing the light-cone gauge on the worldsheet.

The equations of motion are solved by $\hat{S}_L(\tau + \sigma)$, $\hat{S}_R(\tau - \sigma)$ and $Y^i = Y_L^i(\tau + \sigma) + Y_R^i(\tau - \sigma)$ subject to boundary conditions that follow from (4.2).

This action is invariant under the full set of supersymmetry transformations generated by spinors $\hat{\eta}_L(\tau + \sigma)$, $\hat{\eta}_R(\tau - \sigma)$ in the $\mathbf{8}_s$ and the spinors $\hat{\epsilon}_{L,R}$ in the $\mathbf{8}_c$ of $SO(8)$ subject to boundary conditions (see for example [36]). Explicitly, one has the flat background kinematic supersymmetries,

$$\delta_\eta \hat{S}_L = \hat{\eta}_L, \quad \delta_\eta \hat{S}_R = \hat{\eta}_R, \quad (4.7)$$

and the dynamical supersymmetries ($i = 1, 2; a = 3, \dots, 8$),

$$\delta_\epsilon \hat{S}_R = -(\partial_- Y^i \gamma_i + \partial_- X^a \gamma_a) \hat{\epsilon}_R, \quad \delta_\epsilon \hat{S}_L = (\partial_+ Y^i \gamma_i + \partial_+ X^a \gamma_a) \hat{\epsilon}_L, \quad (4.8)$$

$$\delta_\epsilon Y^i = 2i(\hat{\epsilon}_R^T \gamma^i \hat{S}_R - \hat{\epsilon}_L^T \gamma^i \hat{S}_L), \quad \delta_\epsilon X^a = 2i(\hat{\epsilon}_R^T \gamma^a \hat{S}_R - \hat{\epsilon}_L^T \gamma^a \hat{S}_L). \quad (4.9)$$

The boundary conditions (4.2) along with the invariance of the action (4.6) under the above supersymmetry transformations lead to the following boundary conditions on the redefined flat-space quantities,

$$\partial_\tau Y^i|_{\sigma=0,\pi} = 0, \quad \hat{S}_L - \hat{\Omega} \hat{S}_R|_{\sigma=0,\pi}, \quad \hat{\eta}_L - \hat{\Omega} \hat{\eta}_R|_{\sigma=0,\pi}, \quad \hat{\epsilon}_L - \hat{\Omega} \hat{\epsilon}_R|_{\sigma=0,\pi}, \quad (4.10)$$

where $\hat{\Omega} = \exp(-m\sigma\gamma^{12}/2)\Omega\exp(m\sigma\gamma^{12}/2)$. It is easy to verify that the supersymmetry transformations (4.9) keep the boundary conditions invariant provided $\Omega = \hat{\Omega} = 1$.

Thus from the worldsheet point of view, the D-string in the NS5 pp-wave background has as many space-time supersymmetries as the D-string in flat space. However, when written in terms of the variables X^i and $S_{L,R}$, it becomes evident that all these supersymmetry transformations acquire a dependence on the worldsheet coordinate σ in such a way that they do not contain σ -independent zero-mode pieces. Explicitly, the supersymmetry transformations in terms of the original variables become,

$$\delta_\eta S_L = \eta_L, \quad (4.11)$$

$$\delta_\eta S_R = \eta_R, \quad (4.12)$$

$$\delta_\epsilon S_R = -(\partial_- X^i \gamma^i + \partial_- X^a \gamma_a - m \epsilon_{ij} X^i \gamma^j) \epsilon_R, \quad (4.13)$$

$$\delta_\epsilon S_L = (\partial_+ X^i \gamma^i + \partial_+ X^a \gamma_a + m \epsilon_{ij} X^i \gamma^j) \epsilon_L, \quad (4.14)$$

$$\delta_\epsilon X^i = 2i(\epsilon_R^T \gamma^i S_R - \epsilon_L^T \gamma^i S_L), \quad (4.15)$$

which are compatible with the boundary conditions (4.2). The transformation parameters η and ϵ are related to $\hat{\eta}$ and $\hat{\epsilon}$ by equations similar to (4.5). That these do not contain σ -independent pieces (zero-modes along the string) is easy to see. For example, before imposing the boundary condition, $\eta_{L,R}$ have zero modes,

$$(\eta_L)_0 = \frac{1}{2}(1 - i\gamma^{12})\hat{\eta}_{L,-\frac{m}{2}}e^{-im\tau/2} + \frac{1}{2}(1 + i\gamma^{12})\hat{\eta}_{L,\frac{m}{2}}e^{im\tau/2}, \quad (4.16)$$

$$(\eta_R)_0 = \frac{1}{2}(1 - i\gamma^{12})\hat{\eta}_{R,\frac{m}{2}}e^{im\tau/2} + \frac{1}{2}(1 + i\gamma^{12})\hat{\eta}_{R,-\frac{m}{2}}e^{-im\tau/2}, \quad (4.17)$$

where $\hat{\eta}_n$ are the oscillators in the expansion of $\hat{\eta}$. But for the boundary condition $\eta_L - \eta_R|_{\sigma=0,\pi} = 0$ to hold at all τ , one must have,

$$\begin{aligned} (1 + i\gamma^{12})\hat{\eta}_{L,\frac{m}{2}} &= (1 - i\gamma^{12})\hat{\eta}_{R,\frac{m}{2}}, \\ (1 - i\gamma^{12})\hat{\eta}_{L,-\frac{m}{2}} &= (1 + i\gamma^{12})\hat{\eta}_{R,-\frac{m}{2}}, \end{aligned}$$

It is easy to see that the solutions to these equations, when substituted back in (4.16) and (4.17), lead to

$$(\eta_L)_0 = 0, \quad (\eta_R)_0 = 0. \quad (4.18)$$

Thus the supersymmetry transformations that are consistent with the boundary conditions do not have zero modes. In other words, this means that the supercharges do not contain σ -independent components and therefore cannot be expressed in terms of space-time variables alone. Consequently the worldsheet supersymmetries of D1-brane in NS5 pp-wave background are not manifest in the space-time description, consistent with the space-time supersymmetry analysis. Similar situations were encountered in [9] in more involved setups. The example considered here is the simplest and most drastic realization of the phenomenon.

In fact, situations where worldsheet symmetries do not lead to space-time symmetries due the σ -dependence of charges (or equivalently, absence of zero modes in the transformation) have been known for a long time, mostly in relation to T-duality. An example where symmetries and supersymmetries at the current algebra level do not have a space-time realization was discussed in [33] and its relation to T-duality was clarified in [34]. The more general situation where supersymmetry transformations could acquire a σ -dependence as a result of T-duality was analyzed in [35]. In all these cases the symmetry exists at the worldsheet level but is not locally realized on the space-time fields.

5 Summary and Discussion

We have constructed Dp-brane solutions in the background of a pp-wave limit of the linear dilaton geometry. The configurations are such that the light-cone directions of the pp-wave fall within the worldvolume of the D-brane and the directions transverse to the pp-wave are also transverse to the brane. It is shown that these classical solutions do not preserve any space-time supersymmetry.

Such branes can also be constructed in the worldsheet theory in the pp-wave background. In the light-cone gauge, this can be mapped to a theory of strings in flat space-time by simple field redefinitions. It is shown that in the worldsheet description, the D-branes preserve as much supersymmetry as the corresponding D-branes in flat space-time. The contradiction between the space-time and worldsheet results is resolved by showing that all Fourier modes of the allowed supersymmetry parameters depend on the worldsheet coordinate σ , while a local description in terms of space-time

fields should be blind to the extension of the string. As a result it is not possible to express any mode of the supersymmetry parameters in terms of the space-time variables alone, hence the absence of ordinary space-time supersymmetry. This is similar to certain situations discussed in [9] and in fact provides the simplest and most drastic example of this effect. More generally, this is an example of a phenomenon encountered earlier in the context of T-duality [33, 34, 35].

String theory in the linear dilaton pp-wave background is argued to provide a holographic description of a certain high-energy and large R-charge sector of the associated little string theory [20]. The spectrum in this sector was argued to be similar to the spectrum of closed strings in flat 10-dimensional space based on the similarity between the latter and closed strings on the linear dilaton background. However, our discussion shows that this similarity does not extend to string spectrum in the presence of D-branes in a straightforward way. It is interesting to investigate the consequence of this for the defect CFT analogue the little string theory.

Our results can be compared with earlier work on D-branes in the pp-wave limit of $AdS_3 \times S^3$ geometry with NS-NS 3-form background [12, 28]. In this case the structure of the NS-NS 3-form is such that the classical solutions preserve some supersymmetries. An inspection of the worldsheet theory along the lines discussed in section 4 shows that the same feature of the NS-NS 3-form also insures that some of the worldsheet fermions have zero modes resulting in the observed space-time supersymmetries. The fate of the remaining supersymmetries is as discussed above.

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